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Light Entanglement from a Non-Degenerate Three-Level Laser with a Parametric Amplifier and Coupled to a Thermal Reservoir

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Abstract: The quantum properties of a non-degenerate three-level laser with the parametric amplifier and coupled to a thermal reservoir are thoroughly analyzed with the use of the pertinent master equation and stochastic differential equations associated with the normal ordering. Applying solutions of resulting differential equations, quadrature variance, the mean and variance of photon number, the photon number correlation are calculated. However, the two-mode driving light has no effect on the squeezing properties of the cavity modes. Employing the same solutions, one can also obtain anti normally ordered characteristic function defined in the Heisenberg picture. For a linear gain coefficient of (A = 100), for a cavity damping constant of K= 0:8, μ = 0 and for thermal reservoir $\bar{n}$ th = 0, the maximum intra cavity photon entanglement is found at steady state and at threshold to be 60%.

Keywords: Master Equation, Solution of Stochastic Differential Equations, Entanglement Amplification and Langavian Equation

1. Introduction

Three-level lasers have been an interesting area of research over the years in light of its capability to produce radiations with various quantum properties [1-10]. The non-classical states of light (squeezed states) are characterized by a reduction of quantum fluctuations (noise) in one quadrature component of light below the vacuum level, or below that achievable in a coherent state, at the expense of increased fluctuations in the other component such that the product of these fluctuations still obeys the uncertainty relation. Squeezed light has potential applications in low-noise communications and precision measurements [11, 12]. A parametric oscillator has been considered as an important source of squeezed light. It is one of the most interesting and well characterized optical devices in quantum optics. In a cascade three-level laser, three level atoms in a cascade configuration are injected into a cavity coupled to a thermal reservoir via a single-port. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, the two photons are generated as shown in figure 1 below. In this device a pump photon interacts with a nonlinear crystal inside a cavity and is down-converted into two highly correlated photons. If these photons have same frequency the device is called a degenerate parametric oscillator, otherwise it is called a non-degenerate parametric oscillator. The quantum fluctuations and photon statistics of signal mode produced by a non-degenerate parametric oscillator coupled to a two-mode thermal reservoir have been analyzed employing the pertinent Fokker Planck equation or the quantum Langevin equations. The quantum dynamics of a non-degenerate parametric oscillator coupled to a thermal reservoirs have been analyzed employing the Q function obtained by solving the Fokker-Planck equation using the propagator method [13]. When two particles, such as a pair of photons or electrons, become entangled, they remain connected even when separated by vast distances (quantum Entanglement). A two mode sub harmonic generator at the lower and above threshold has been theoretically predicted to be a source of light in an entangled state [14]. Recently, the experimental realization of the entanglement in two-mode sub harmonic generator has been demonstrated by Zhang et al. [15]. On the other hand, Xiong et al. [16] have recently proposed a scheme for an entanglement based on a non-degenerate three -level laser can atoms are injected at the lower level and the top levels are coupled by a strong coherent light. They have found that a non-degenerate three level laser can generate light in an entangled state employing the entanglement criteria for bipartite continuous variables states. Moreover, Tan et al. [17] have extended the work of Xiong et al. and examined the generation and evolution of entangled light in the Wigner representation using the sufficient and necessary in separability criteria for a two -mode Gaussian state proposed by Duan et al. [18] and Simon [19]. The generation and manipulation of entanglement has attracted a great deal of interest owing to their wide applications in quantum teleportation [20], quantum dense coding [21], quantum computation [22], quantum error correction [23], and quantum cryptography [24]. The variance of the quadrature operators and the photon number distribution for the signal-idler modes Producing by generation of entanglement from non-degenerate three level laser with parametric oscillation have also been studied applying the pertinent Langevin equations. One can first obtain stochastic differential equations for the cavity mode variables by applying the pertinent Master equation [25-28]. With the aid of resulting equations, quadrature variance for the two-mode cavity radiation and the squeezing are calculated. In addition, one can determine the mean photon number, the photon number entanglement, and the variance of the photon number difference, and the photon number correlation. The mean, the variance, and the photon number correlation, in the absence of the parametric amplifier (μ = 0) also calculated.

2. Master Equation

The equation of evolution of density operator for the three-level laser applying the linear and the adiabatic approximation schemes first derived [4-7]. Then, one can represent the top, intermediate, and bottom levels of a three-level atom in a cascade configuration by $|a\rangle, |b\rangle$ and $|c\rangle$, respectively, as shown in Figure 1.

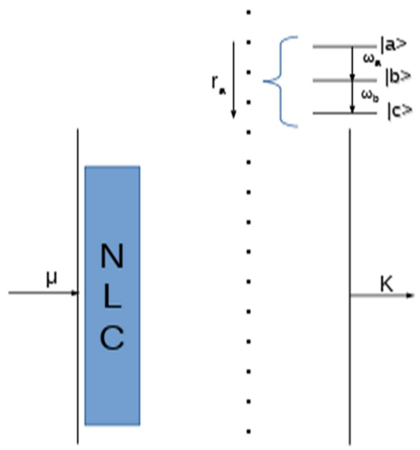

Figure 1. Schematic representation of a non-degenerate three level laser with a parametric amplifier and a thermal reservoir.

This figure shows the Entangled Photon Generation from a Three-Level Laser with a Parametric Amplifier and coupled to a thermal reservoir. In addition, we assume the two modes a and b to be at resonance with the two transitions |a⟩ to |b⟩ and |a⟩ to |c⟩ dipole allowed respectively, and direct transition between levels |a⟩ to |c⟩ to be dipole forbidden. The interaction of non-degenerate three-level atom with the cavity modes can be described by the Hamiltonian.

$$\hat{H}_1 = ig[|a\rangle\langle b|\hat{a} - \hat{a}^+|b\rangle\langle a| + |b\rangle\langle c|\hat{b} - \hat{b}^+|c\rangle\langle b|. \quad (1)$$

Where g is the coupling constant and $\hat{a}(\hat{b})$ is the annihilation operators for the cavity modes. Moreover, the Hamiltonian describing the parametric interaction [8-11], with the pump mode treated classically, can be written as

$$\hat{H}_2 = i\mu\left(\hat{a}^+\hat{b}^+ - \hat{a}\hat{b}\right). \quad (2)$$

In which μ is proportional to the amplitude of the pump mode [12-14]. Here, taking the initial state of a single three-level atom and hence, the density operator of a single atom is

$$\hat{\rho}_A(0) = \rho^{(0)}{}_{aa}|a\rangle\langle a| + \rho^{(0)}{}_{ac}|a\rangle\langle c| + \rho^{(0)}{}_{ca}\,|c\rangle\langle a| + \rho^{(0)}{}_{cc}\,|c\rangle\langle c|. \quad (3)$$

Moreover, employing Eq. 1, the master equation for the cavity modes coupled to thermal reservoir, put in the form.

$$\frac{d}{dt}\hat{\rho}(t) = -i\left[\hat{H}_{S,\hat{\rho}}\right] + g(\rho_{ab}\hat{a}^+ - \hat{a}^+\rho_{ab}) + \rho_{bc}\hat{b}^+ - \hat{b}^+\rho_{bc} + \hat{a}\rho_{ba-\rho_{ba}\hat{a}} + \hat{b}\rho_{cb} - \rho_{cb}\hat{b})$$
$$+\frac{K}{2}(\bar{n}th+1)(2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}) + \frac{K}{2}\bar{n}th(2\hat{a}^+\hat{\rho}\hat{a} - \hat{a}\hat{a}^+\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^+)$$
$$+\frac{K}{2}(\bar{n}th+1)\left(2\hat{b}\hat{\rho}\hat{b}^+ - \hat{b}^+\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^+\hat{b}\right) + \frac{K}{2}\bar{n}th\left(2\hat{b}^+\hat{\rho}\hat{b} - \hat{b}\hat{b}^+\hat{\rho} - \hat{\rho}\hat{b}\hat{b}^+\right). \quad (4)$$

In which the matrix element $\rho_{\alpha\beta}$ is defined by

$$\rho_{\alpha\beta} = \langle\alpha|\hat{\rho}_{AR}|\beta\rangle, \quad (5)$$

With α, β = a, b, c. Using once more the adiabatic approximation scheme, we see that

$$\hat{\rho}_{ab} = \frac{gr_a}{\gamma^2}\left(\rho^{(0)}{}_{ac}\hat{\rho}\hat{b}^+ - \rho^{(0)}{}_{aa}\hat{\rho}\hat{a}\right), \quad (6)$$

$$\hat{\rho}_{bc} = \frac{gr_a}{\gamma^2}\left(\rho^{(0)}{}_{cc}\hat{\rho}\hat{b}^+ - \rho^{(0)}{}_{ac}\hat{a}^+\hat{\rho}\right). \quad (7)$$

Finally, on account of Eqs. (6), and (7), the equation of evolution of the density operator for the cavity modes given by Eq. (4), takes the form.

$$\frac{d}{dt}\hat{\rho}\,(t) = -i[\hat{H}_2,\hat{\rho}] + \frac{K}{2}(\bar{n}th+1)[2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}] + \frac{1}{2}\left(A\rho^{(0)}{}_{aa} + K\hat{n}th\right)[2\hat{a}^+\hat{\rho}\hat{a} - \hat{a}\hat{a}^+\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^+]$$
$$+\frac{1}{2}\left(A\rho^{(0)}{}_{cc} + K(\bar{n}th+1)\right)[2\hat{b}\hat{\rho}\hat{b}^+ - \hat{b}^+\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^+\hat{b}] + \frac{1}{2}K\bar{n}th[2\hat{b}^+\hat{\rho}\hat{b} - \hat{b}\hat{b}^+\hat{\rho} - \hat{\rho}\hat{b}\hat{b}^+]$$
$$+\frac{1}{2}A\rho^{(0)}{}_{ac}[\hat{a}\hat{a}\hat{\rho} - \hat{a}^+\hat{\rho}\hat{b}^+ + \hat{\rho}\,\hat{a}^+\hat{b}^+ - \hat{b}\hat{\rho}\hat{a}] + \frac{1}{2}A\rho^{(0)}{}_{ca}[\hat{a}^+\hat{b}^+\hat{\rho} - \hat{a}^+\hat{\rho}\,\hat{b}^+ + \hat{\rho}\hat{a}\hat{b} - \hat{b}\hat{\rho}\hat{a}]. \quad (8)$$

Where,

$$A = \frac{2r_{ag^2}}{\gamma^2}. \quad (9)$$

Is linear gain coefficient. The equation of evolution of the density operator associated with the Hamiltonian given by Eq. (2) has the form.

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{2}\mu[\hat{\rho}\hat{a}\hat{b} - \hat{a}\hat{b}\hat{\rho} + \hat{a}^+\hat{b}^+\hat{\rho} - \hat{\rho}\,\hat{a}^+\hat{b}^+] + \frac{K}{2}(\bar{n}th+1)[2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}]$$
$$+\frac{1}{2}\left(A\rho^{(0)}{}_{aa} + K\bar{n}th\right)[2\hat{a}^+\hat{\rho}\hat{a} - \hat{a}\hat{a}^+\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^+] + \frac{1}{2}\left(A\rho^{(0)}{}_{cc} + K(\bar{n}th+1)\right)[2\hat{b}\hat{\rho}\hat{b}^+ - \hat{b}^+\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^+\hat{b}]$$
$$+\frac{1}{2}K\bar{n}\,[2b^+\hat{\rho}\hat{b} - \hat{b}\hat{b}^+\hat{\rho} - \hat{\rho}\hat{b}\hat{b}^+] + \frac{1}{2}A\rho^{(0)}{}_{ac}[\hat{a}\hat{b}\hat{\rho} - \hat{a}^+\hat{\rho}\hat{b}^+ + \hat{\rho}\hat{a}^+\hat{b}^+ - \hat{b}\hat{\rho}\hat{a}]$$
$$+\frac{1}{2}A\rho^{(0)}{}_{ca}[\hat{a}^+\hat{b}^+\hat{\rho} - \hat{a}^+\hat{\rho}\hat{b}^+ + \hat{\rho}\hat{a}\hat{a} - \hat{b}\hat{\rho}\hat{a}]. \quad (10)$$

This is the master equation for the cavity modes of a non-degenerate three-level laser whose cavity contains a non –degenerate parametric amplifier and coupled to a thermal reservoir.

A. The Stochastic Differential equations

Next we seek to determine the solutions of the stochastic differential equations. Thus employing

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d}{dt}\hat{\rho}(t)\hat{A}\right). \quad (11)$$

Along with Eq. 11, and applying the cyclic property of the trace operation together with the commutation relations

$$[\hat{a},\hat{a}^+] = [\hat{b},\hat{b}^+] = 1, \quad (12)$$

And

$$[\hat{a},\hat{a}] = [\hat{b},\hat{b}] = [\hat{a},\hat{b}] = 0. \quad (13)$$

We readily obtain

$$\frac{d}{dt}\langle\hat{a}\rangle = -\frac{\mu_a}{2}\langle\hat{a}\rangle + \frac{1}{2}v_-\langle\hat{b}^+\rangle, \quad (14)$$

$$\frac{d}{dt}\langle\hat{b}\rangle = -\frac{\mu_c}{2}\langle\hat{b}\rangle + \frac{1}{2}v_+\langle\hat{a}^+\rangle, \quad (15)$$

$$\frac{d}{dt}\langle\hat{a}^2\rangle = -\mu_a\langle\hat{a}^2\rangle + v_-\langle\hat{b}^+\hat{a}\rangle, \quad (16)$$

$$\frac{d}{dt}\langle\hat{b}^2\rangle = -\mu_c\langle\hat{b}^2\rangle + v_+\langle\hat{a}^+\hat{b}\,\rangle, \quad (17)$$

$$\frac{d}{dt}\langle\hat{a}^+\hat{a}\rangle = -\mu_a\langle\hat{a}^+\hat{a}\rangle + \frac{1}{2}v_-\langle\hat{a}^+\hat{b}^+\rangle + \frac{1}{2}v^*{}_-\langle\hat{a}\hat{b}\rangle + A\rho^{(0)}{}_{aa} + K\bar{n}, \quad (18)$$

$$\frac{d}{dt}\langle\hat{b}^+\hat{b}\rangle = -\mu_c\langle\hat{b}^+\hat{b}\rangle + \frac{1}{2}v_+\langle\hat{b}^+\hat{a}^+\rangle + \frac{1}{2}v^*{}_+\langle\hat{a}\hat{b}\rangle + k\bar{n}, \quad (19)$$

$$\frac{d}{dt}\langle\hat{a}^+\hat{b}\rangle = -\frac{1}{2}(\mu_a+\mu_c)\langle\hat{a}^+\hat{b}\rangle + \frac{1}{2}v_+\langle\hat{a}^{+2}\rangle + \frac{1}{2}v^*{}_-\langle\hat{b}^{+2}\rangle, \quad (20)$$

$$\frac{d}{dt}\langle\hat{a}\hat{b}\rangle = -\frac{1}{2}(\mu_a+\mu_c)\langle\hat{a}\hat{b}\rangle + \frac{1}{2}v_+\langle\hat{a}^+\hat{a}\rangle + \frac{1}{2}v_-\langle\hat{b}^+\hat{b}\rangle. \quad (21)$$

Where,

$$\mu_a = K - A\rho^{(0)}{}_{aa}, \tag{22}$$

$$\mu_c = K + A\rho^{(0)}{}_{cc}, \tag{23}$$

$$v_{\pm} = 2\mu \pm A\rho^{(0)}{}_{ac}. \tag{24}$$

We note that the corresponding c- numbers are

$$\frac{d}{dt}\langle\alpha\rangle = -\frac{\mu_a}{2}\langle\alpha\rangle + \frac{1}{2}v_-\langle\beta^*\rangle, \tag{25}$$

$$\frac{d}{dt}\langle\beta\rangle = -\frac{\mu_c}{2}\langle\beta\rangle + \frac{1}{2}v_+\langle\alpha^*\rangle, \tag{26}$$

$$\frac{d}{dt}\langle\alpha^2\rangle = -\mu_a\langle\alpha^2\rangle + v_-\langle\beta^*\alpha\rangle, \tag{27}$$

$$\frac{d}{dt}\langle\beta^2\rangle = -\mu_c\langle\beta^2\rangle + v_+\langle\alpha^*\beta\rangle, \tag{28}$$

$$\frac{d}{dt}\langle\alpha^*\alpha\rangle = -\mu_a\langle\alpha^*\alpha\rangle + \frac{1}{2}v_-\langle\alpha^*\beta^*\rangle + \frac{1}{2}v^*{}_-\langle\alpha\beta\rangle + A\rho^{(0)}{}_{aa} + K\bar{n}, \tag{29}$$

$$\frac{d}{dt}\langle\beta^*\beta\rangle = -\mu_c\langle\beta^*\beta\rangle + \frac{1}{2}v_+\langle\beta^*\alpha^*\rangle + \frac{1}{2}v^*{}_+\langle\alpha\beta\rangle + A\rho^{(0)}{}_{cc} + K\bar{n}, \tag{30}$$

$$\frac{d}{dt}\langle\alpha^*\beta\rangle = -\frac{1}{2}(\mu_a + \mu_c)\langle\alpha^*\beta\rangle + \frac{1}{2}v_+\langle\alpha^{*2}\rangle + \frac{1}{2}v^*{}_-\langle\beta^{*2}\rangle, \tag{31}$$

$$\frac{d}{dt}\langle\alpha\beta\rangle = -\frac{1}{2}(\mu_a + \mu_c)\langle\alpha\beta\rangle + \frac{1}{2}v_+\langle\alpha^*\alpha\rangle + \frac{1}{2}v_-\langle\beta^*\beta\rangle. \tag{32}$$

On basis of Eqs. (25), and (26), we can write

$$\frac{d}{dt}\alpha(t) = -\frac{\mu_a}{2}\langle\alpha\rangle + \frac{1}{2}v_-\langle\beta^*\rangle + f_\alpha(t), \tag{33}$$

$$\frac{d}{dt}\beta^*(t) = -\frac{\mu_c}{2}\langle\beta\rangle + \frac{1}{2}v_+\langle\alpha^*\rangle + f^*{}_\beta(t). \tag{34}$$

Where fα (t) and fβ (t) are noise forces. We now proceed to determine the properties of the noise force. The expectation value of Eqs. (33) and (34) are found to be

$$\frac{d}{dt}\langle\alpha(t)\rangle = -\frac{\mu_a}{2}\langle\alpha\rangle + \frac{1}{2}v_-\langle\beta^*\rangle + \langle f_\alpha(t)\rangle, \tag{35}$$

$$\frac{d}{dt}\langle\beta^*(t)\rangle = -\frac{\mu_c}{2}\langle\beta\rangle + \frac{1}{2}v_+\langle\alpha^*\rangle + \langle f_\beta(t)\rangle. \tag{36}$$

Comparison of Eqs. (25) and (35) as well as Eqs. (26) and (36) yields

$$\langle f_\alpha(t)\rangle = \langle f_\beta(t)\rangle = 0. \tag{37}$$

The formal solutions of Eqs. (36) and (37) can be put in the form

$$\alpha(t) = \alpha(0)e^{-\frac{\mu_a t}{2}} + \int_0^t e^{-\frac{\mu_a(t-t')}{2}}\left[\frac{1}{2}v_-\beta^*(t') + f_\alpha(t')\right]dt', \tag{38}$$

$$\beta^*(t) = \beta(0)e^{-\frac{\mu_c}{2}t} + \int_0^t e^{-\frac{\mu_c(t-t')}{2}}\left[\frac{1}{2}v^*{}_+\alpha(t') + f^*{}_\beta(t')\right]dt'. \tag{39}$$

Moreover, applying the relation

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = \langle\alpha(t)\left(\frac{d}{dt}\alpha(t)\right)\rangle + \langle\left(\frac{d}{dt}\alpha(t)\right)\alpha(t)\rangle. \tag{40}$$

Along with Eq. (35), one can readily verify that

$$\frac{d}{dt}\langle\alpha^2\rangle = -\mu_a\langle\alpha^2(t)\rangle + v^*{}_-\langle\alpha(t)\rangle + 2\langle\alpha(t)f_\alpha(t)\rangle. \tag{41}$$

With aid of Eq. (36), one can readily verify that using the same relation

$$\langle\beta^*(t)f_\alpha(t)\rangle = 0. \tag{42}$$

In view of this result, one can readily get

$$\int_0^t e^{\frac{\mu_{a(t-t')}}{2}} \langle f_\alpha(t')f_\alpha(t)\rangle dt'. \tag{43}$$

Applying the relation

$$\int_0^t e^{\frac{1a(t-t')}{2}} \langle f(t)g(t')\rangle dt' = D, \tag{44}$$

We assert that

$$\langle f(t)g(t')\rangle = 2D\delta(t-t'),$$

Where a and D are a constants or some function of time t. We then see that

$$\langle f_\alpha(t')f_\alpha(t)\rangle = 0. \tag{45}$$

It can also be established in similar manner that

$$\langle f_\beta(t')f_\beta(t)\rangle = \langle f^*{}_\alpha(t')f_\beta(t)\rangle = 0. \tag{46}$$

With Eq. (35) and its complex conjugate, we have

$$\frac{d}{dt}\langle \alpha^*\alpha\rangle = -\mu_a\langle \alpha^*\alpha\rangle + \frac{1}{2}v_-\langle \alpha^*\beta^*\rangle + \frac{1}{2}v^*{}_-\langle \alpha\beta\rangle + \langle \alpha^*(t)f_\alpha(t)\rangle + \langle f_\alpha{}^*(t)\alpha(t)\rangle. \tag{47}$$

3. Quadrature Variance

Here seeking to analyze the quadrature squeezing properties of the two-mode light in the cavity can be described by two quadrature's [15-16].

$$\hat{c}_+ = \hat{c}^+ + \hat{c}, \tag{48}$$

$$\hat{c}_- = i(\hat{c}^+ - \hat{c}). \tag{49}$$

Where,

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b}), \tag{50}$$

$$\hat{c}^+ = \frac{1}{\sqrt{2}}(\hat{a}^+ + \hat{b}^+). \tag{51}$$

Are the two-mode cavity operators, $\hat{a}$ and $\hat{b}$ are annihilation operators for cavity modes a and b. In view of Eq. (50) and Eq. (51), one can write Eq. (48) as

$$\hat{c}_+ = \frac{1}{\sqrt{2}}\left[\hat{a} + \hat{b} + \hat{a}^+ + \hat{b}^+\right]. \tag{52}$$

It then follows that

$$\hat{c}_+ = \frac{1}{\sqrt{2}}\left[\hat{a}_+ + \hat{b}_+\right]. \tag{53}$$

Following a similar procedure, we get

$$\hat{c}_- = \frac{i}{\sqrt{2}}\left[\hat{a}_- + \hat{b}_-\right]. \tag{54}$$

Where,

$$\hat{a}_+ = \hat{a} + \hat{a}^+, \hat{a}_- = i(\hat{a}^+ - \hat{a}), \tag{55}$$

$$\hat{b}_+ = \hat{b} + \hat{b}^+, \hat{b}_- = i(\hat{b}^+ - \hat{b}). \tag{56}$$

Employing the commutation relation of the cavity mode operators

$$[\hat{a}, \hat{a}^+] = [\hat{b}, \hat{b}^+] = 1. \tag{57}$$

The quadrature operators $\hat{c}_+$ and $\hat{c}_-$ are Hermitian and satisfy the commutation relation

$$[\hat{c}_+, \hat{c}_-] = 2i. \tag{58}$$

The variance of the plus and minus quadrature operators of the two-mode cavity light are defined by

$$(\Delta\hat{c}_+)^2 = \langle \hat{c}^2{}_+\rangle - \langle \hat{c}_+\rangle^2, \tag{59}$$

And

$$(\Delta\hat{c}_-)^2 = \langle \hat{c}_-^2\rangle - \langle \hat{c}_-\rangle^2. \tag{60}$$

On account of Eqs. (48) and (59), the plus quadrature variance can be expressed in terms of the creation and annihilation operators as

$$(\Delta\hat{c}_+)^2 = \langle \hat{c}\hat{c}^+\rangle + \langle \hat{c}^+\hat{c}\rangle + \langle \hat{c}^2\rangle + \langle \hat{c}^{+2}\rangle - \langle \hat{c}\rangle^2 - \langle \hat{c}^+\rangle^2 - 2\langle \hat{c}\rangle\langle \hat{c}^+\rangle, \tag{61}$$

And with the help of Eqs. (49) and (60), we get

$$(\Delta\hat{c}_-)^2 = \langle \hat{c}\hat{c}^+\rangle + \langle \hat{c}^+\hat{c}\rangle - \langle \hat{c}^2\rangle - \langle \hat{c}^{+2}\rangle + \langle \hat{c}\rangle^2 + \langle \hat{c}^+\rangle^2 - 2\langle \hat{c}\rangle\langle \hat{c}^+\rangle. \tag{62}$$

So that inspection of Eqs. (61) and (62) shows that

$$(\Delta\hat{c}_\pm)^2 = \langle \hat{c}\hat{c}^+\rangle + \langle \hat{c}^+\hat{c}\rangle \pm \langle \hat{c}^2\rangle \pm \langle \hat{c}^{+2}\rangle \mp \langle \hat{c}\rangle^2 \mp \langle \hat{c}^+\rangle^2 - 2\langle \hat{c}\rangle\langle \hat{c}^+\rangle. \tag{63}$$

This can be expressed in terms of c-number variables associated with the normal ordering as

$$(\hat{c}_{\pm})^2 = \langle\gamma(t)\gamma^*(t)\rangle + \langle\gamma^*(t)\gamma(t)\rangle \pm \langle\gamma^2(t)\rangle \pm \langle\gamma^{*2}(t)\rangle \mp \langle\gamma(t)\rangle^2 \mp \langle\gamma^*(t)\rangle^2 - 2\langle\gamma(t)\rangle\langle\gamma^*(t)\rangle \tag{64}$$

Where γ (t) is the c-number variable corresponding to the operator $\hat{c}$(t). The c-number equation corresponding to Eq. (50) can be written as

$$\gamma(t) = \frac{1}{\sqrt{2}}[\alpha(t) + \beta(t)]. \tag{65}$$

And application of Eq. (65) to Eq. (64) leads to

$$\Delta\hat{c}_{\pm}^2 = 1 \pm [\tfrac{1}{2}\langle\alpha^2(t)\rangle + \langle\alpha^{*2}(t)\rangle + \langle\beta^2(t)\rangle + \langle\beta^{*2}(t)\rangle + \langle\alpha(t)\beta(t)\rangle + \langle\alpha^*(t)\beta^*(t)\rangle \pm \langle\alpha^*(t)\alpha(t)\rangle$$

$$+\langle\beta^*(t)\beta(t)\rangle + \langle\alpha^*(t)\beta(t)\rangle + \langle\beta^*(t)\alpha(t)\rangle] \mp \frac{1}{2}\langle(\alpha^*(t) + \beta^*(t) \pm \alpha(t) + \beta(t))\rangle^2. \tag{66}$$

Assuming that the cavity modes are initially in vacuum state along with the fact that a noise force at a certain time does not affect the cavity mode variables at earlier time [17-19], we easily find

$$\langle\alpha^2(t)\rangle = 0, \tag{67}$$

In a similar manner, we see that

$$\langle\beta^2(t)\rangle = 0, \tag{68}$$

$$\langle\alpha^*(t)\beta(t)\rangle = \langle\beta^*(t)\alpha(t)\rangle = 0. \tag{69}$$

Now with the aid of Eqs. (67), (68), and (69), we arrive at

$$\Delta c_{\pm}^2 = 1 + \frac{1}{2}[\langle\alpha(t)\beta(t)\rangle + \langle\alpha^*(t)\beta^*(t)\rangle] \pm \langle\alpha^*(t)\alpha(t)\rangle + \langle\beta^*(t)\beta(t)\rangle. \tag{70}$$

Since $\langle\alpha(t)\beta(t)\rangle = \langle\alpha^*(t)\beta^*(t)\rangle$, we then see that

$$\Delta c_{\pm}^2 = 1 + \langle\alpha(t)\beta(t)\rangle \pm \langle\alpha^*(t)\alpha(t)\rangle + \langle\beta^*(t)\beta(t)\rangle. \tag{71}$$

This takes the form

$$\Delta {c^2}_{\pm} = 1 + \frac{2KA(1-\eta)(2K+2A\eta+A)+16\mu^2 A\eta-4KA\eta^2\bar{n}th}{4[K(k+A\eta)-4\mu^2](2K+A\eta)} \pm \frac{2K(4\mu+A\sqrt{1-\eta^2}(2K+A\eta+A\pm 4\mu)}{4[K(k+A\eta)-4\mu^2](2K+A\eta)}$$

$$+\frac{4K[(2K+A\eta)(2K+A\eta\pm 4\mu)\bar{n}n+A^2(1\pm\sqrt{1-\eta^2})\bar{n}th]}{4[K(k+A\eta)-4\mu^2](2K+A\eta)}. \tag{72}$$

This represents the quadrature variances of the cavity modes for a non-degenerate three level laser whose cavity contains a parametric amplifier and coupled to a thermal reservoir.

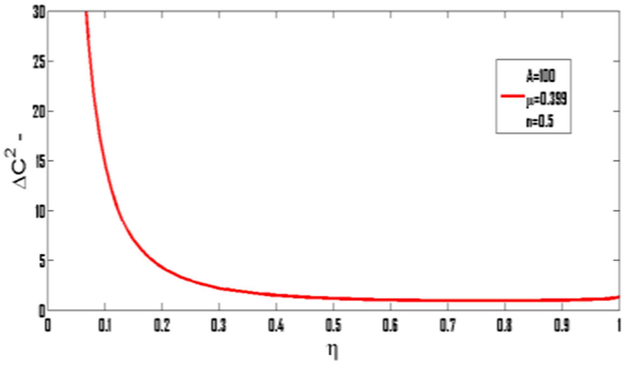

Figure 2. *The quadrature variances versus η for A = 100, κ = 0:8, μ = 0:399, and $\bar{n}$th = 0:5.*

Plot in Figure 2 indicates that the maximum intra cavity squeezing for the above values and within the parametric amplifier is 50% below the coherent state level. Figure 2 is the plot of variance of the minus quadrature versus η with parametric amplifier in non-degenerate three-level laser cavity.

Next upon setting $\bar{n}$th = μ = 0 in Eq. (72), we get

$$\Delta\hat{c}_{\pm}^2 = 1 + \frac{A(1-\eta)(2K+2A\eta+A)-A\sqrt{1-\eta^2}\,(2K+A\eta+A)}{2(K+A\eta)(2K+A\eta)}. \tag{73}$$

This is the quadrature variances of the cavity modes for a non-degenerate three-level laser.

In Figure 3 the minimum value of the quadrature variance described by Equation. (73) for A = 100, k = 0:8, and $\bar{n}$th = μ = 0 is found to be Δc_{-}^2= 0.45 and occurs at η = 0:16. This result implies that the maximum intra cavity squeezing for the above values is 40% below the coherent-state level. The plots in Figure 3 represent the variances of the minus quadrature of the cavity modes for a non-degenerate three-level laser alone.

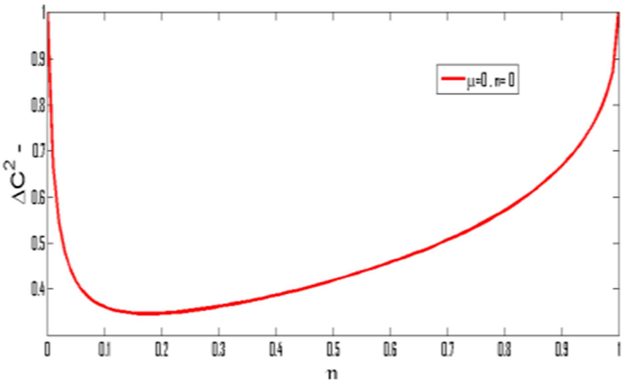

Figure 3. *The quadrature variances versus η for A = 100, K = 0:8, μ = 0, and $\bar{n}th$ = 0.*

4. Photon Statistics

A. The mean and the variance of the photon number

The mean photon number for the two-modes in terms of density operator can be expressed as

$$\langle \hat{c}^{+}(t)\hat{c}(t)\rangle = Tr\big(\rho(t)\hat{c}^{+}(0)\hat{c}(0)\big), \tag{74}$$

In which

$$\hat{c} = \hat{a} + \hat{b}, \tag{75}$$

$$\hat{c}^{+} = \hat{a}^{+} + \hat{b}^{+}. \tag{76}$$

Where $\hat{a}, \hat{b}$ *and* $\hat{c}$ are the annihilation operators for a light mode a, light mode b, and the two-mode light, respectively [20-24]. Employing Eqs. (73) And (74), Eq. (72) can be written as

$$\langle \hat{c}^{+}(t)\hat{c}(t)\rangle = \langle \hat{a}^{+}(t)\hat{a}(t)\rangle + \langle \hat{a}^{+}(t)\hat{b}(t)\rangle + \langle \hat{b}^{+}(t)\hat{a}(t)\rangle + \langle \hat{b}^{+}\hat{b}(t)\rangle. \tag{77}$$

Employing the relation

$$\int d^2\alpha e^{-a\alpha^{*}\alpha + b\alpha + c\alpha^{*}} = \frac{\pi}{a} e^{\frac{bc}{a}}, \tag{78}$$

With performing the integration over λ, it yields

$$\langle \hat{a}^{+}(t)\hat{a}(t)\rangle = \frac{1}{\pi^4}(u^2 - v^2)\frac{d^2}{dxdy}\int d^2\alpha d^2\beta d^2\eta \exp(-\eta^{*}\eta + \eta^{*}(\alpha + y)$$

$$+\eta(\alpha^{*} + v\beta - v\alpha *) \exp(-\alpha^{*}\alpha + x\alpha + v\alpha^{*}\beta^{*} - u\beta^{*}\beta)\, | x = y = 0. \tag{79}$$

So that carrying out the integration over β and η, there follows

$$\langle \hat{a}^{+}(t)\hat{a}(t)\rangle = \frac{1}{\pi^4}(u^2 - v^2)\int d^2\alpha \exp(-\alpha^{*}\alpha(\frac{u^2-v^2}{u}) + \alpha^{*}(uy + u^2y + v^2y) + x\alpha)|\, x = y = 0. \tag{80}$$

Performing differentiation, by applying the condition, x = y = 0, we readily obtain

$$\langle \hat{a}^{+}(t)\hat{a}(t)\rangle = a - 1. \tag{81}$$

Similarly, following the same procedure, we note that

$$\langle \hat{b}^{+}(t)\hat{b}(t)\rangle = b - 1. \tag{82}$$

Now we see that

$$\bar{n} = \frac{2K\big(4\mu + A\sqrt{1-\eta^2}\big)(2K + A\eta + A - 4\mu)}{4[K(K + A\eta) - 4\mu^2](2K + A\eta)} + \frac{4K[(2K + A\eta)(2K + A\eta + 4\mu)\bar{n}_{th} + A^2(1 - \sqrt{1-\eta^2})\bar{n}_{th}]}{4[K(K + A\eta) - 4\mu^2](2K + A\eta)}. \tag{83}$$

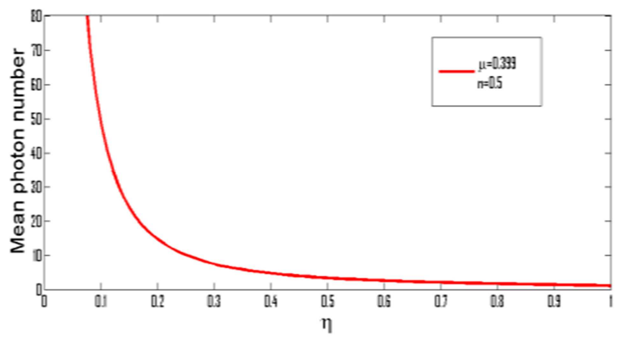

Figure 4. *The mean photon number versus η for A = 100, κ = 0:8, μ = 0:399, and $\bar{n}th$ = 0:5.*

The plot on Figure 4 shows that the mean photon number of Eq. (83) for the values A = 100, κ = 0:8, μ = 0:399, and $\bar{n}th$ = 0.5. The results show that as η increases the mean photon number decreases.

Finally, in the absence of both parametric amplifier (when μ = 0) and thermal reservoir (when $\bar{n}th$ = 0), the mean photon number of Eq. (83) turns out to be

$$\bar{n} = \frac{(A\sqrt{1-\eta^2})(2K + A\eta + A)}{2(K + A\eta)(2K + A\eta)}. \tag{84}$$

Figure 5 shows that the plot of mean photon number in the absence of both parametric amplifier and thermal reservoir for the values A = 100, κ = 0:8, μ = 0, and $\bar{n}th$ = 0. The plot in Figure 5 shows that the mean photon number decrease as η increases.

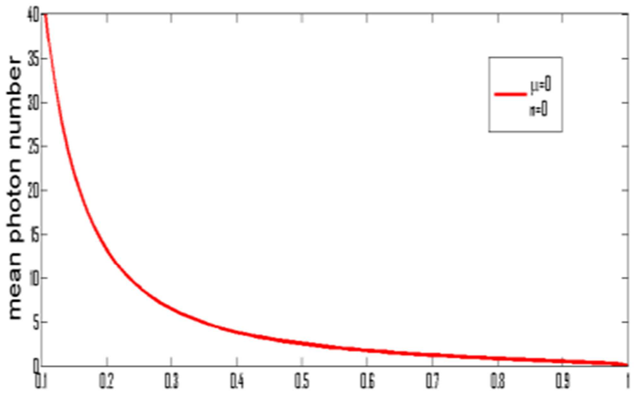

Figure 5. *The mean photon number versus η for A = 100, κ = 0:8, μ = 0, and n̄th = 0.*

B. The Variance of the Photon Number Difference

The variance of the photon number at steady state can be expressed as

$$(\Delta n)^2 = \langle(\hat{c}^{+}\hat{c})^2\rangle - \langle\hat{c}^{+}\hat{c}\rangle^2. \quad (85)$$

Where,

$$\hat{c} = \hat{a} + \hat{b},$$
$$\hat{c}^{+} = \hat{a}^{+} + \hat{b}^{+}. \quad (86)$$

And

$$\gamma = \alpha + \beta,$$
$$\gamma^{*} = \alpha^{*} + \beta^{*}. \quad (87)$$

Are c-number variables associated with the normal ordering. The Photon number variance takes the form

$$(\Delta n)^2 = \langle\hat{c}^{+}\hat{c}\rangle\langle\hat{c}\hat{c}^{+}\rangle + \langle\hat{c}^2\rangle\langle\hat{c}^{+2}\rangle, \quad (88)$$

From which follows

$$(\Delta n)^2 = 2[1 + \langle\hat{c}^{+}\hat{c}\rangle] + \langle\hat{c}^2\rangle\langle\hat{c}^{+2}\rangle. \quad (89)$$

It is possible to write in c-number as

$$(\Delta n)^2 = 2[1 + \langle(\alpha(t) + \beta(t))(\alpha^{*}(t) + \beta^{*}(t))\rangle] + \langle\left(\alpha(t)\beta(t)\right)^2\rangle + \langle(\alpha^{*}(t) + \beta^{*}(t))\rangle. \quad (90)$$

With the aid of

$$\langle\alpha^{*}(t)\beta^{*}(t)\rangle = \langle\alpha(t)\beta(t)\rangle. \quad (91)$$

One can verify that

$$(\Delta n)^2 = 2[1 + \langle\alpha^{*}(t)\alpha(t)\rangle + \langle\beta^{*}(t)\beta(t)\rangle + 2\langle\alpha(t)\beta(t)\rangle]. \quad (92)$$

Thus the variance of the photon number takes the form

$$(\Delta n)^2 = 2 + 2\left[\frac{2K(4\mu + A\sqrt{1-\eta^2})(2K+A\eta+A-4\mu)}{4[K(K+A\eta)-4\mu^2](2K+A\eta)}\right] + 2\left[\frac{4K[(2K+A\eta)(2K+A\eta+4\mu)\bar{n}_{th}+A^2(1-\sqrt{1-\eta^2})\bar{n}_{th}]}{4[K(K+A\eta)-4\mu^2](2K+A\eta)}\right] + 4\left[\frac{2KA(1-\eta)(2K+2A\eta+A)+16\mu^2A\eta-4KA^2\eta^2\bar{n}}{4[K(K+A\eta)-4\mu^2](2K+A\eta)}\right]. \quad (93)$$

This is the photon number variance for a coherently driven three-level laser with parametric amplifier

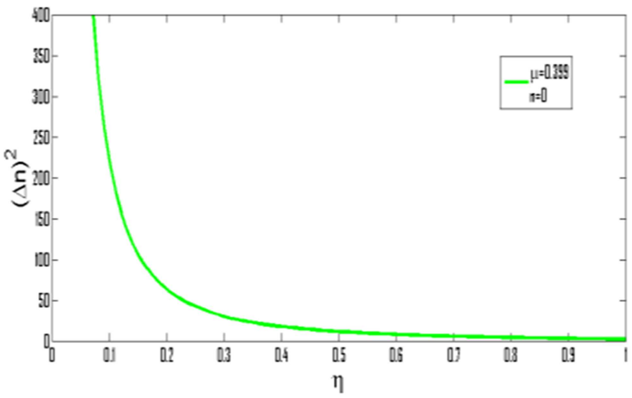

Figure 6. *The variance of photon number difference versus η for A = 100, κ = 0:8, μ = 0:399, and n̄th = 0.*

Figure 6 shows that the plot of photon number variance in the absence of thermal reservoir for the values A = 100, κ = 0:8, μ = 0:399, and n̄th = 0. The plot in Figure 6 shows that the variance of photon number decrease as η increases.

Furthermore, in the absence of both parametric amplifier (when μ = 0) and thermal reservoir (when n̄th = 0, the variance of the photon number described by Eq. (93) reduces to

$$(\Delta n)^2 = 2 + 2\left[\frac{2K(A\sqrt{1-\eta^2})(2K+A\eta+A)}{4[K(K+A\eta)](2K+A\eta)}\right]$$
$$+4\left[\frac{2KA(1-\eta)(2K+2A\eta+A)}{4[K(K+A\eta)](2K+A\eta)}\right]. \quad (94)$$

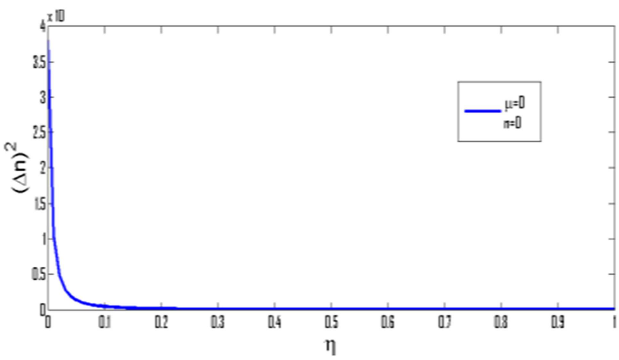

Figure 7. *The variance of photon number difference versus η for A = 100, κ = 0:8, μ = 0, and n̄th = 0.*

Figure 7 shows that the plot of photon number variance in the absence of both parametric amplifier and thermal reservoir for the values A = 100, κ = 0:8, μ = 0, and $\bar{n}$th = 0. The plot in Figure 7 shows that the variance of photon number decrease as η increases.

5. Entaglement Amplification

Here the entanglement condition of the two modes in the cavity was studied. A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its individual constituents. The preparation and manipulation of these entangled states that have non-classical and nonlocal properties lead to a better understanding of the basic quantum principles [25-28]. That is, if the density operator for the combined state cannot be described as a combination of the product of density operators of the constituents.

$$\hat{\rho} \neq \sum_j p_j \hat{\rho}_j^{(1)} \otimes \hat{\rho}_j{}^{(2)}\,, \tag{95}$$

Where $\otimes = bigtimes\,, p_j \geq 0\ and\ \sum_j p_j = 1$ is set to ensure normalization of the combined density of state.

To study the properties of entanglement produced by this quantum optical system, we need an entanglement criterion for the system. According to the criteria set by Duan et al. [20], a quantum state of the system is entangled provided that the sum of the variances of the two EPR (Einstein-Podolsky-Rosen)-type operators (entanglement) $\hat{u}$ and $\hat{v}$ sat isfies the condition;

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 < 2. \tag{96}$$

Where,

$$\hat{u} = \hat{x}_a - \hat{x}_b, \hat{v} = \hat{p}_a + \hat{p}_b, \tag{97}$$

With

$$\hat{x}_a = \frac{(\hat{a}^+ + \hat{a})}{\sqrt{2}}, \hat{x}_b = \frac{(\hat{b}^+ + \hat{b})}{\sqrt{2}}, \tag{98}$$

$$\hat{p}_a = \frac{i(\hat{a}^+ - \hat{a})}{\sqrt{2}}, \hat{p}_b = \frac{i(\hat{b}^+ - \hat{b})}{\sqrt{2}}. \tag{99}$$

Being the quadrature operators for modes $\hat{a}$ and $\hat{b}$. The total variance of the operators ^ u and ^ v can be written as

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 < 2. \tag{100}$$

This implies that

$$(\Delta\hat{u})^2 = \langle u^2\rangle - \langle u\rangle^2. \tag{101}$$

On account of Eq. 101, we see that

$$(\Delta\hat{u})^2 = \langle(\tfrac{1}{2}(\hat{a} + \hat{a}^+)\rangle - \langle\tfrac{1}{2}(\hat{b} + \hat{b}^+)^2)\rangle. \tag{102}$$

From which follows

$$(\Delta\hat{u})^2 = \tfrac{1}{2}[1 + 2\langle\hat{a}^+\hat{a}\rangle] - \tfrac{1}{2}[2\langle\hat{a}\hat{b}\rangle] + \tfrac{1}{2}[1 + 2\langle\hat{b}^+\hat{b}\rangle]. \tag{103}$$

It then follows that

$$(\Delta\hat{u})^2 = 1 + 2\langle\hat{a}^+\hat{a}\rangle + 2\langle\hat{b}^+\hat{b}\rangle - 2\langle\hat{a}\hat{b}\rangle. \tag{104}$$

It is possible to write Eq. (104), in case of c-number variables.

$$(\Delta\hat{u})^2 = [1 + 2\langle\alpha^*(t)\alpha(t)\rangle + 2\langle\beta^*(t)\beta(t)\rangle - 2\langle\alpha(t)\beta(t)\rangle\,], \tag{105}$$

Following the same procedure, we easily obtain

$$(\Delta\hat{v})^2 = [1 + 2\langle\alpha^*(t)\alpha(t)\rangle + 2\langle\beta^*(t)\beta(t)\rangle - 2\langle\alpha(t)\beta(t)\rangle]. \tag{106}$$

Thus, the sum of the variances of u and v can be expressed as

$$(\Delta\hat{u})^2 + (\Delta v)^2 = 2(\Delta\hat{u})^2 = 2(\Delta\hat{c}_{\pm})^2. \tag{107}$$

From this result that the degree of entanglement is directly proportional to the degree of squeezing of the two-mode light. Therefore, we see that

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = 2[1 + 2\langle\alpha^*(t)\alpha(t)\rangle + 2\langle\beta^*(t)\beta(t)\rangle - 2\langle\alpha(t)\beta(t)\rangle]. \tag{108}$$

This can be rewritten as

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = 2 + 2\langle\alpha^*(t)\alpha(t)\rangle + 2\langle\beta^*(t)\beta(t)\rangle - 4\langle\alpha(t)\beta(t)\rangle. \tag{109}$$

In view of Eqs. (73), (74), and (75), Eq. (108) takes the form

$$(\Delta\hat{\boldsymbol{u}})^2 + (\Delta\hat{v})^2 = 2 + 2\left[\frac{2K\left(4\mu + A\sqrt{1-\eta^2}\right)(2K + A\eta + A - 4\mu)}{4[k(k+A\eta) - 4\mu^2](2K + A\eta)}\right] + 2\left[\frac{4K\left[(2K+A\eta)(2K+A\eta+4\mu)(\bar{n}_{th}) + A^2\left(1-\sqrt{1-\eta^2}\right)(\bar{n}_{th})\right]}{4[k(k+A\eta) - 4\mu^2\,](2K+A\eta)}\right]$$

$$+4\left[\frac{2KA(1-\eta)(2K+2A\eta+A) + 16\mu^2 A\eta - 4KA^2\eta^2\bar{n}_{th}}{4[k(k+A\eta) - 4\mu^2\,](2K+A\eta)}\right]. \tag{110}$$

Considering the case in which the parametric amplifier is removed from the cavity. Thus setting μ = 0 in Eq. (110), one can readily verify that

$$(\Delta\hat{u})^2 + (\hat{v})^2 = 2\left[1+\frac{A(1-\eta)(2K+1A\eta)-2A^2\eta^2\bar{n}_{th}}{2(k+A\eta)(2K+A\eta)}\right] \pm \left[\frac{A\sqrt{1-\eta^2}(2K+A\eta+A+2A\bar{n}_{th})}{2(k+A\eta)(2K+A\eta)}+\frac{[(2K+A\eta)^2\bar{n}+A^2\bar{n}_{th}]}{2(k+A\eta)(2K+A\eta)}\right]. \quad (111)$$

This represents the photon entanglement of the cavity modes for a non-degenerate three level lasers coupled to a two-mode squeezed vacuum reservoir.

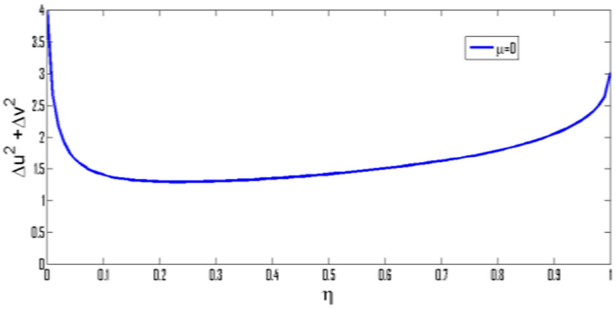

Figure 8. *$\Delta\hat{u}^2 + \Delta\hat{v}^2$ of two-mode light in the cavity at steady state versus η for κ = 0:8, A = 100, μ = 0, and $\bar{n}$th = 0:5.*

The minimum value of the photon entanglement is found to be $\Delta U^2 + \Delta v^2$ = 0:144 and occurs at η = 0:1. For A = 100, κ = 0:8, μ = 0, and $\bar{n}$th = 0:5. This indicates that the maximum intra cavity squeezing for the above values and in the absence of parametric amplifier is 90% below the coherent state level. Figure 8 is the plots of the photon entanglement versus η in the absence of parametric amplifier in non-degenerate three-level laser cavity. This figure shows that the increase of the degree of squeezing due to the parametric amplifier is not significant.

Now consider the case in which the nonlinear crystal is removed from the cavity and the cavity is coupled to a two-mode vacuum reservoir. Then upon setting μ = $\bar{n}$ = 0 in Eq. (110), we get

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = 2\left[1+\frac{A(1-\eta)(2K+2A\eta+A)-A\sqrt{1-\eta^2}(2K+A\eta+A)}{2(k+A\eta)(2K+A\eta)}\right]. \quad (112)$$

This is the photon entanglement of the cavity modes of a non-degenerate three-level laser with vacuum reservoir.

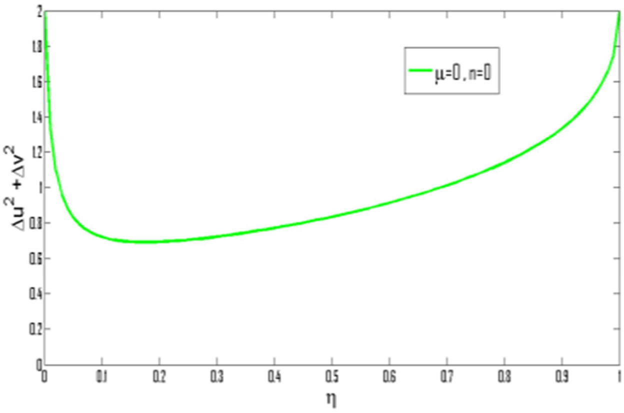

Figure 9. *$\Delta\hat{u}^2 + \Delta\hat{v}^2$ Of two-mode light in the cavity at steady state versus η for κ = 0:8, A = 100, μ = 0, and $\bar{n}$th = 0.*

The minimum value of the photon entanglement described by (112) for A = 100, k = 0.8, μ = 0 and $\bar{n}$ = 0 is found to be 70% and occurs at η = 0:16. This result implies that the maximum intracavity squeezing for the above values is 75% below the coherent-state level. The plots in Figure 9 represent the photon entanglement of the cavity modes for a non-degenerate three-level laser alone.

6. Conclusion

In this article, the squeezing, entanglement, and statistical properties of the light produced by a non-degenerate three-level laser coupled to a two-mode thermal reservoir have been analyzed in the linear and adiabatic approximation schemes in the good cavity limit. Then using the master equation, stochastic differential equations was obtained. Applying the solutions of the resulting differential equations, the quadrature variance was calculated. Employing the solutions for the c-number cavity mode variables along with the correlation property of noise forces associated with a normal ordering, the quadrature squeezing, photon entanglement, mean number of photon numbers are obtained. Increasing the amplitude of the parametric amplifier increases the mean photon numbers and the variances of the photon numbers have also been founded. The effect of the squeezed vacuum is to enhance the degree of squeezing of the signal-idler modes was observed. Furthermore, the mean photon number of mode a is greater than that of mode b have been resulted. Both the mean photon number and the quadrature variance for the two-mode laser light beams are the sum of the mean photon numbers and the quadrature variances of the constituent two-mode laser light beams had been founded. Therefore, the increase in the mean photon number is observed in a region, where the degrees of two-mode squeezing and entanglement are significant making the system under consideration available source of intense squeezed, as well as entangled, light.

Conflict of Interest

No conflict of interest from the author.

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